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Building Home Plate: Field of Dreams or Reality?

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In the movie Field of Dreams, Kevin Costner's character, Ray Kinsella, considers building a baseball park in the middle of his cornfield. "If you build it, they will come," encourages a voice from the past. As an assistant coach for my nine-year-old son's baseball team, I was interested to read in the official league rules the following specifications for home plate:

"Home base shall be marked by a five-sided slab of whitened rubber. It shall be a 12-inch square with two of the corners filled in so that one edge is 17 inches long, two are 8 1/2 inches and two are 12 inches." ([1], p. 160)

An accompanying diagram shows the finished product:

Pondering these instructions, I wondered not whether we should build it, but whether we could build it. Is such a home plate possible?

The "correct" answer is "No." The figure implies the existence of a right isosceles triangle with sides 12, 12 and 17. But (12, 12, 17) is not (quite) a Pythagorean triple: $12^2 + 12^2 = 288$; $17^2 = 289$. Thus, these specifications seem to give new meaning to a "Field of Dreams."
On the other hand, if one interprets the values 12 and 17 as measured numbers, accurate to two significant digits, then home plate can be built, since, to that degree of accuracy, (12, 12, 17) is a Pythagorean triple.

We can build it! Let them come.

REFERENCE

E. S. Loomis, in [1], argues that there can be no trigonometric proof nor any proof based on analytical geometry or calculus for the Pythagorean proposition because each of these subjects "accepts the truth of geometry as established, and therefore furnishes no new proof." His argument seems valid in so far as functions of two (or more) variables are involved in such a 'proof.' Since calculus of one variable can be developed without using the Pythagorean theorem, circular reasoning may be avoided. The following is a proof of the proposition using calculus.

Let $ABC$ be a triangle with its right angle at $A$. Keep $AB$ fixed and let $AB = b$. Denote $AC$ by the variable $x$, so that $BC$ is a function of $x$, $f(x)$. See Figure 1. If $AC$ increases by an amount $\Delta x$, then $BC$ will increase by $\Delta f$. From similar triangles,

\[
\frac{\Delta f}{\Delta x} = \frac{CQ}{CD} > \frac{CP}{CD} = \frac{CA}{CB} = \frac{x}{f(x)}.
\]